

1 Information Technology and Electronics, Entrance Exam 18.02.2022; Mathematics & Physics

The duration of the exam is three (3) hours. Return your solutions in one pdf-file. In total there is seven problems in this exam (Max score $7 \cdot 10 = 70$ points).

1. Solve the following equations:
 - (a) $x^2 + 5x - 6 = 0$,
 - (b) $10^{x^2+1} = 100000$,
 - (c) $\cos(x) = 0.5$.
2. Solve the following inequalities:
 - (a) $2x + 1 > x + 2$,
 - (b) $\frac{x+1}{2x-6} \leq 1$.
3. Machines M and N manufacture components. The probability that a component is of an acceptable standard is 0.9 when manufactured by machine M and 0.8 when manufactured by machine N . Machine M supplies 65 % of the components and machine N supplies the rest.
 - (a) Calculate the probability that a component picked at random is of an acceptable standard.
 - (b) A component is not of an acceptable standard. Calculate the probability that it is made by machine N .
 - (c) Two components are picked at random. Calculate the probability that they are made by different machines.
4. The Wimbledon tennis championship is a single-elimination tournament where a player is eliminated after a single loss. There is 128 players in the men's championship. In year 2017 Roger Federer won the title.
 - (a) How many matches did Roger play?
 - (b) How many matches were totally played in men's tournament?
5. A rectangle with sides parallel to the coordinate axes is inscribed in the circle

$$x^2 + y^2 = 1.$$

Find the largest possible area for this rectangle.

6. A kid (mass 25 kg) is at the top of the hill (height is 7.2 m). Then he/she starts to slide down. Assume that hill is frictionless.
- (a) What is the initial potential energy?
 - (b) What is the velocity at the bottom of hill?
 - (c) At the bottom is a level but rough stretch where the coefficient of kinetic friction is 0.25 . How far does he/she slide across the level stretch?
7. The voltage of a capacitor decreases exponentially

$$U(t) = 5V \cdot e^{-\frac{t}{T}},$$

where T is the time constant. We know that $U(3s) = 2V$ (the physical unit are seconds s and volts V).

- (a) Find the value of T .
- (b) When will the voltage be $1V$?